Task 2

Introduction to the measuring system PULSE and measurement of self sustained oscillations of metal rod

- Familiarize yourself with the system PULSE from Brüel & Kjær
- Measure impulse response of steel beam generated by force impact
- Measurement should be performed for excitation in three points and three position of accelerometers. Compare measuremets with theory.
- Report should contain analyzed data, no theoretical introduction is required.

2.1 Theory

Let us have a rod of rectangular cros-section with length l, clamped at one side. This rod is excited by small tap creating oscilating movement. We assume bending waves only (no longitudial or torsional waves are discussed here). For this type of waves we can derive following equation of motion (see eg. [1]):

$$\frac{\partial^2 u}{\partial t^2} + \frac{Eh^2}{12\rho}\frac{\partial^4 u}{\partial z^4} = 0, \tag{2.1}$$

where E is Young modulus, u is displacement, h is a thickness of the rod, ρ is density, t is time and z is coordinate oriented parallel to the longitudial axcis of the rod.

We are looking for the solution in the form of product of two independant functions of one variable:

$$u(z,t) = \psi(t)\varphi(z) \tag{2.2}$$

After some tedious algebra we find the solution in the form

$$u(z,t) = \sum_{n=1}^{\infty} \varphi_n (F_n \cos\omega_n t + G_n \sin\omega_n t), \tag{2.3}$$

where

$$\varphi_n = A_n \left[(\cos m_n + \cosh m_n)\left(\cos\frac{m_n}{l}z - \cosh\frac{m_n}{l}z\right) + \right. \\ \left. + (\sin m_n - \sinh m_n)\left(\sin\frac{m_n}{l}z - \sinh\frac{m_n}{l}z\right)\right]. \tag{2.4}$$

We can determine constants F_n and G_n from the initial conditions (that describe deflection at the time $t = 0$) and constants A_n from the regularization conditions (modal deformations are orthogonal, therefore inner products of two different modes are zero, while inner product of the same modes lead to the A_n).

Boundary conditions for one-side clamped rod are

$$\varphi(0) = 0, \quad \left(\frac{d\varphi}{dz}\right)_{z=0} = 0, \quad \left(\frac{d^2\varphi}{dz^2}\right)_{z=l} = 0, \quad \left(\frac{d^3\varphi}{dz^3}\right)_{z=l} = 0$$

Applying theese conditions we obtain transcendent equation for values m_n

$$\cos m \cosh m = -1. \tag{2.5}$$

The first 15 equation roots approximately equals

$$\begin{array}{lll} m_1 = 1{,}8751; & m_2 = 4{,}6941; & m_3 = 7{,}8548; \\ m_4 = 10{,}9955; & m_5 = 14{,}1371; & m_6 = 17{,}2787; \\ m_7 = 20{,}4203; & m_8 = 23{,}5619; & m_9 = 26{,}7035; \\ m_{10} = 29{,}8451; & m_{11} = 32{,}9867; & m_{12} = 36{,}1283; \\ m_{13} = 39{,}2699; & m_{14} = 42{,}4115; & m_{15} = 45{,}5531. \end{array}$$

For angular frequency of modes of the rod ω_n then it follows

$$\omega_n = \frac{m_n^2}{l^2}\sqrt{\frac{Eh^2}{12\rho}}. \tag{2.6}$$

From previous derivation it follows that bending vibrations of the rod in this arrangement are not harmonic multiples of some basic frequency, but their are in the ratio of squares of m_n.

If we take into account real dimensions of the rod we can calculate eigenfrequencies. For the used rod we can assume the Young's modulus $E = 2{,}1 \cdot 10^{11}\,\mathrm{Pa}$ and density $\rho = 8000\,\mathrm{kg/m^3}$.

During the measurement you can observe that positions of accelerometer affect magnitudes of particular modes. It is caused by distribution of nodes and antinodes along the rod. Analyze the measured spectrum of vibration, identify bending modes. Use the figures and discuss the effect of accelerometer position to the measured data.

Literatura

[1] Miroslav Brdička, Ladislav Samek, Bruno Sopko: Mechanika kontinua, Academia, Praha 2000