1 Introduction to physical acoustics

1.1 Specification of branch

- Mechanics of continuous matter – Continuum machanics
  - macroscopic approach – continuous environment
  - element of matter is not atom or melecule
  - exist at least two derivatives of all functions
  - solids and fluids
- Fluids
  - liquis and gases
  - still too wide branch
- Inviscid compressible fluid
  - barotropic fluid
  - e.g. air
  - linear approximation

1.2 Basic equations of linear acoustics

- Equations of motion – Euler hydrodynamic equations

$$\frac{\partial v_i}{\partial t} + \frac{\partial v_i}{\partial x_j} v_j = G_i - \frac{1}{\rho}\frac{\partial p}{\partial x_i}$$

- Continuity equation – conservation of mass

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v_i)}{\partial x_i} = 0, \text{ resp. } \frac{\partial \rho}{\partial t} + \mathrm{div}\ (\rho \boldsymbol{v}) = 0$$

- Equation of state

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{\kappa},$$

- Nonlinear equations – for linear acoustics we need linearization

1.3 Equations of linear acoustics

Linearization conditions

$$\rho = \rho_0 + \rho', \ p = p_0 + p' \text{ a } \boldsymbol{v} = \boldsymbol{v}'$$

$$\frac{\rho'}{\rho_0} \ll 1 \quad \text{a} \quad \frac{p'}{p_0} \ll 1$$

Equations of linear acoustics

$$\rho_0 \frac{\partial v_i'}{\partial t} = -\frac{\partial p'}{\partial x_i}$$

$$\frac{\partial \rho'}{\partial t} + \rho_0 \frac{\partial v_i'}{\partial x_i} = 0$$

$$p' = c_0^2 \rho'$$

After exclusion of equation of state

$$\rho_0 \frac{\partial v_i'}{\partial t} + \frac{\partial p'}{\partial x_i} = 0, \quad c_0^{-2}\frac{\partial p'}{\partial t} + \rho_0 \frac{\partial v_i'}{\partial x_i} = 0$$

2 Wave equation

Wave equation for pressure

1. $(\partial/\partial x_i)$, 2. $(\partial/\partial t)$ and subtract them

$$\frac{\partial^2 p'}{\partial x_i^2} - c_0^{-2}\frac{\partial^2 p'}{\partial t^2} = 0, \text{ resp. div grad } p' - c_0^{-2}\frac{\partial^2 p'}{\partial t^2} = 0$$

$$c_0^2 = \frac{\kappa p_0}{\rho_0}, \text{ tedy } c_0 = \sqrt{\frac{\kappa p_0}{\rho_0}}, \quad c_0 = \sqrt{\frac{\kappa R\Theta}{M}},$$

Solution of wave equation

- One-dimensional wave equation (Jean le Rond d'Alembert)

$$\frac{\partial^2 p}{\partial x^2} - c_0^{-2}\frac{\partial^2 p}{\partial t^2} = 0 \quad p(x,t) = f(c_0 t - x) + g(c_0 t + x)$$

- Plane progressive (traveling) wave

$$f = A\mathrm{e}^{\mathrm{j}k(c_0 t - x)} = A\mathrm{e}^{\mathrm{j}(\omega t - kx)}$$

- Characteristic resistance of medium

$$z_0 = \frac{p}{v} = \frac{\rho_0 \omega}{k} = \rho_0 c_0$$

- Energy flow - sound intensity

$$I = \frac{1}{T}\int_{-T/2}^{T/2} p(t)v(t)\mathrm{d}t = \frac{1}{\rho_0 c_0}\frac{1}{T}\int_{-T/2}^{T/2} p^2(t)\mathrm{d}t = \frac{p_{ef}^2}{\rho_0 c_0}$$

- Standing wave

$$p = A\mathrm{e}^{\mathrm{j}(\omega t - kx)} + B\mathrm{e}^{\mathrm{j}(\omega t + kx)} = \left(A\mathrm{e}^{-\mathrm{j}kx} + B\mathrm{e}^{\mathrm{j}kx}\right)\mathrm{e}^{\mathrm{j}\omega t}$$

$A = B$

$$p = A\left(\mathrm{e}^{-\mathrm{j}kx} + \mathrm{e}^{\mathrm{j}kx}\right)\mathrm{e}^{\mathrm{j}\omega t} = 2A\cos kx\,\mathrm{e}^{\mathrm{j}\omega t}$$

2.1 Energy conservation law in acoustics

Linearized equations after exclusion of equation of state

$$\rho_0\frac{\partial v_i'}{\partial t} + \frac{\partial p'}{\partial x_i} = 0, \qquad \frac{1}{\rho_0 c_0^2}\frac{\partial p'}{\partial t} + \frac{\partial v_i'}{\partial x_i} = 0$$

1. scalar multiply by particle velocity v_i',
2. multiply by sound pressure p' and add togther

$$p'\left(\frac{1}{\rho_0 c_0^2}\frac{\partial p'}{\partial t} + \frac{\partial v_i'}{\partial x_i}\right) + \left(\rho_0\frac{\partial v_i'}{\partial t} + \frac{\partial p'}{\partial x_i}\right)v_i' = 0$$

$$\text{div } (u\boldsymbol{a}) = \boldsymbol{a}\cdot\text{grad } u + u \text{ div } \boldsymbol{a}, \quad \text{resp.} \quad \frac{\partial}{\partial x_i}(u a_i) = a_i\frac{\partial u}{\partial x_i} + u\frac{\partial a_i}{\partial x_i}$$

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\frac{p'^2(t)}{\rho_0 c_0^2} + \frac{1}{2}\rho_0 v_i'(t)v_i'(t)\right] + \frac{\partial}{\partial x_i}\left[p'(t)v_i'(t)\right] = 0$$

2.1.1 Instantaneous sound intensity

$$i_i(t) = p'(t)v_i'(t), \quad \text{resp.} \quad \boldsymbol{i}(t) = p'(t)\boldsymbol{v}'(t)$$

2.1.2 Sound intensity

$$\boldsymbol{I} \equiv \lim_{T\to\infty} \frac{1}{T}\int_0^T \boldsymbol{i}(t)\mathrm{d}t$$

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\frac{p'^2(t)}{\rho_0 c_0^2} + \frac{1}{2}\rho_0 v_i'(t)v_i'(t)\right] + \frac{\partial}{\partial x_i}\left[p'(t)v_i'(t)\right] = 0$$

2.1.3 Sound energy density

$$e(t) = e_k(t) + e_p(t) = \frac{1}{2}\rho_0\left[\boldsymbol{v}'^2(t) + \frac{p'^2(t)}{(\rho_0 c_0)^2}\right]$$

2.1.4 Energy conservation law

$$\frac{\partial e(t)}{\partial t} + \nabla\cdot\boldsymbol{i}(t) = 0$$

(outside sources)

supplied power W'

$$\nabla\cdot\boldsymbol{i}(t) = -\frac{\partial e(t)}{\partial t} + W'$$

After time integration

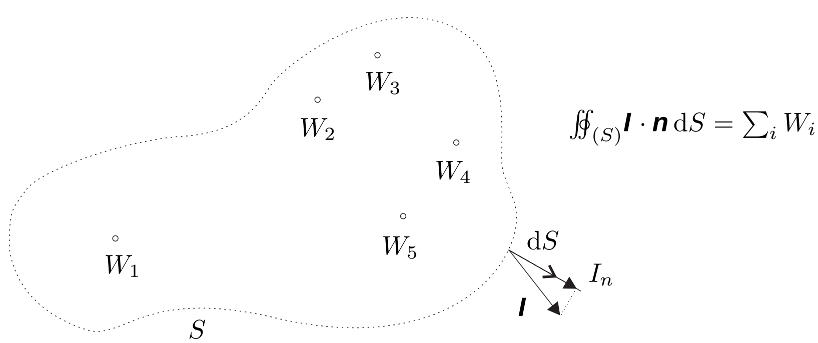
$$\iiint_{(V)} \nabla\cdot\boldsymbol{I}\,\mathrm{d}V = \iiint_{(V)} W'\mathrm{d}V.$$

Using Gauss' theorem

$$\oint\!\!\!\oint_{(S)} \boldsymbol{I}\cdot\boldsymbol{n}\,\mathrm{d}S = W_S,$$

W_S – mean sound power

$$W = \oint\!\!\!\oint_{(S)} \boldsymbol{I}\cdot\boldsymbol{n}\,\mathrm{d}S$$



Option of surface defines the source

2.2 Measurement of sound intensity

$$\boldsymbol{I} \equiv \lim_{T\to\infty} \frac{1}{T}\int_0^T \boldsymbol{i}(t)\mathrm{d}t = \lim_{T\to\infty} \frac{1}{T}\int_0^T p'\boldsymbol{v}'(t)\mathrm{d}t$$

1. Measured p and $\boldsymbol{v}$ – p-v probes
2. Sound pressure in two adjecent points – p-p probes

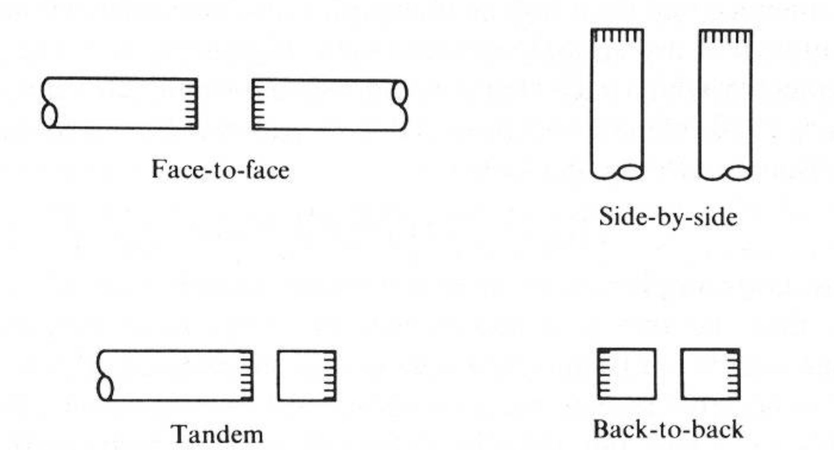
Using Euler equation

$$\rho_0 \frac{\partial v_n}{\partial t} = -\frac{\partial p}{\partial n}$$

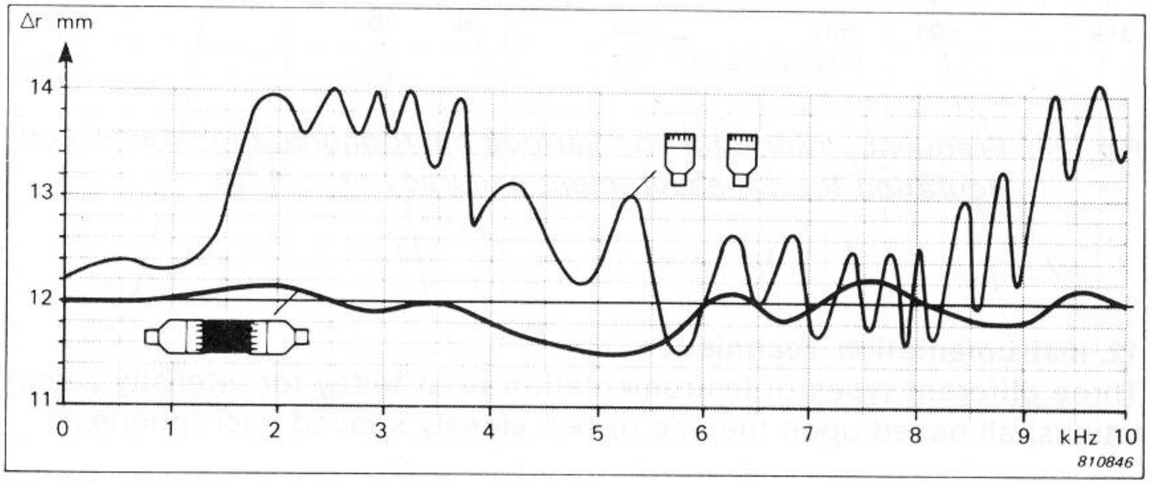
$$I_n \approx \lim_{T\to\infty} \frac{1}{T}\int_0^T \left[\frac{1}{2\rho_0 d}[p_1(t)+p_2(t)]\int_{-\infty}^t [p_1(\tau)-p_2(\tau)]\mathrm{d}\tau\right]\mathrm{d}t$$

$$I_n(f) = -\frac{1}{\rho_0\omega d}\Im\{G_{p_1p_2}(f)\}$$

Arrangement of microphones

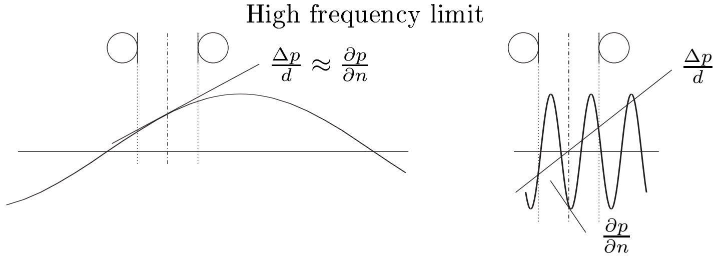


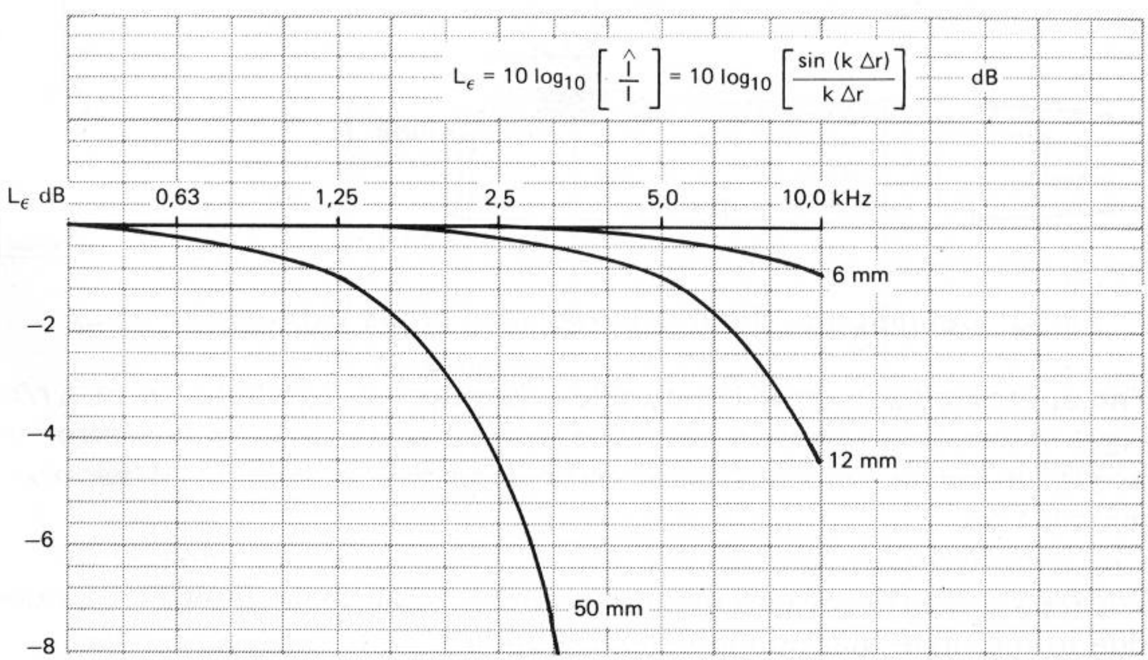
Apparent distance of microphone centers



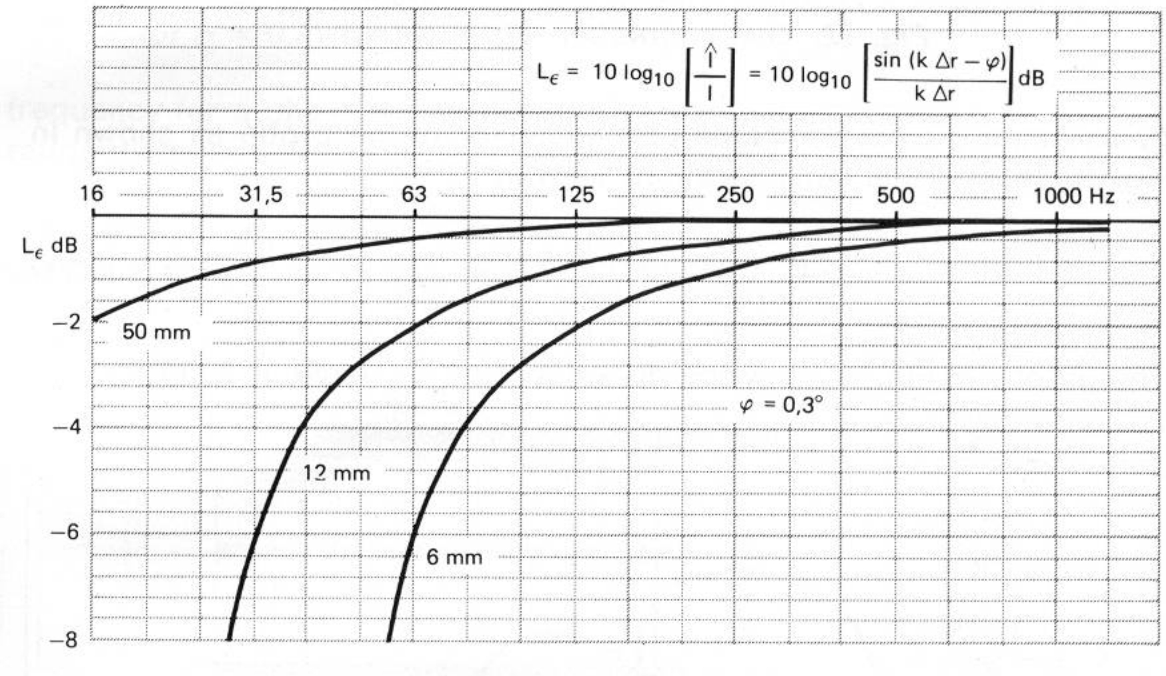
$$I_n \approx \lim_{T\to\infty} \frac{1}{T} \int_0^T \left[\frac{1}{2\rho_0 d} [p_1(t) + p_2(t)] \int_{-\infty}^t [p_1(\tau) - p_2(\tau)] \mathrm{d}\tau \right] \mathrm{d}t$$

High frequency limit

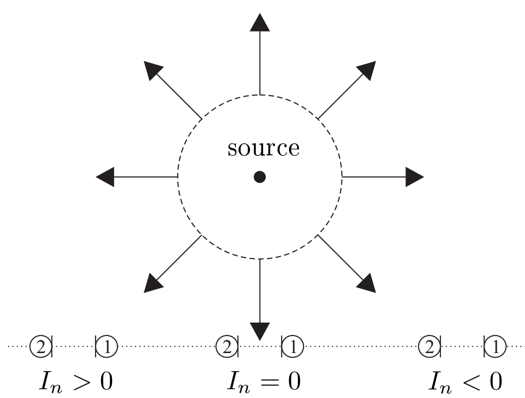


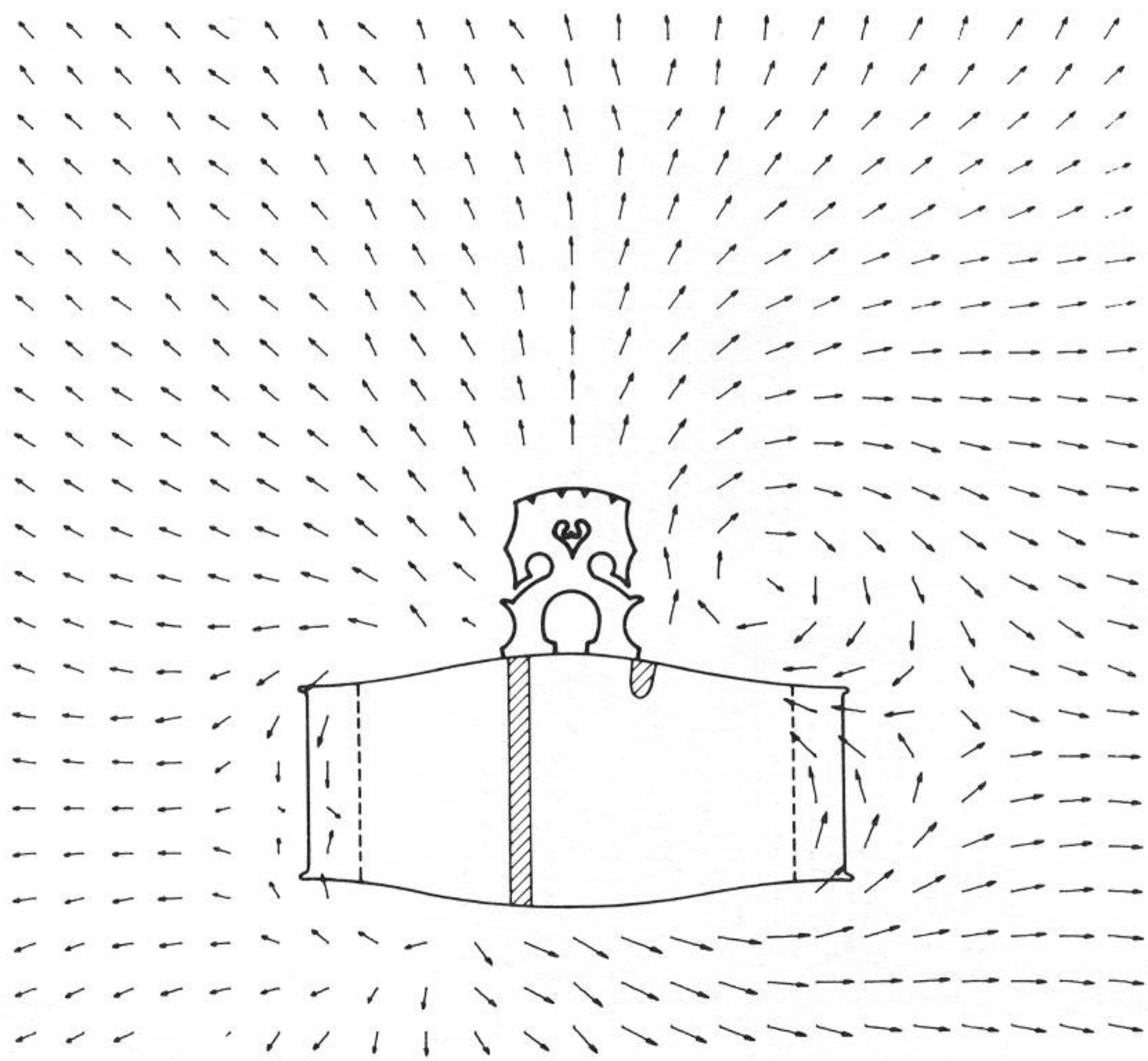


Low frequency limit



Sound source localization





2.3 Sound power measurement

- Measurement in reverberation room (ISO 3741, 3743)
- Measurement in anechoic room (ISO 3745, 3744)

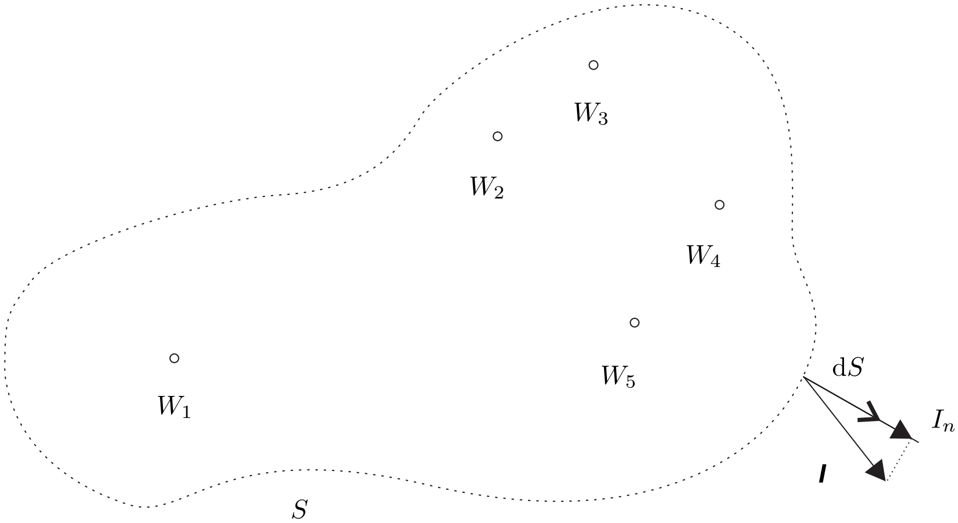
$$W = \oiint_{(S)} \boldsymbol{I} \cdot \boldsymbol{n} \,\mathrm{d}S$$

$$L_p \approx L_I$$

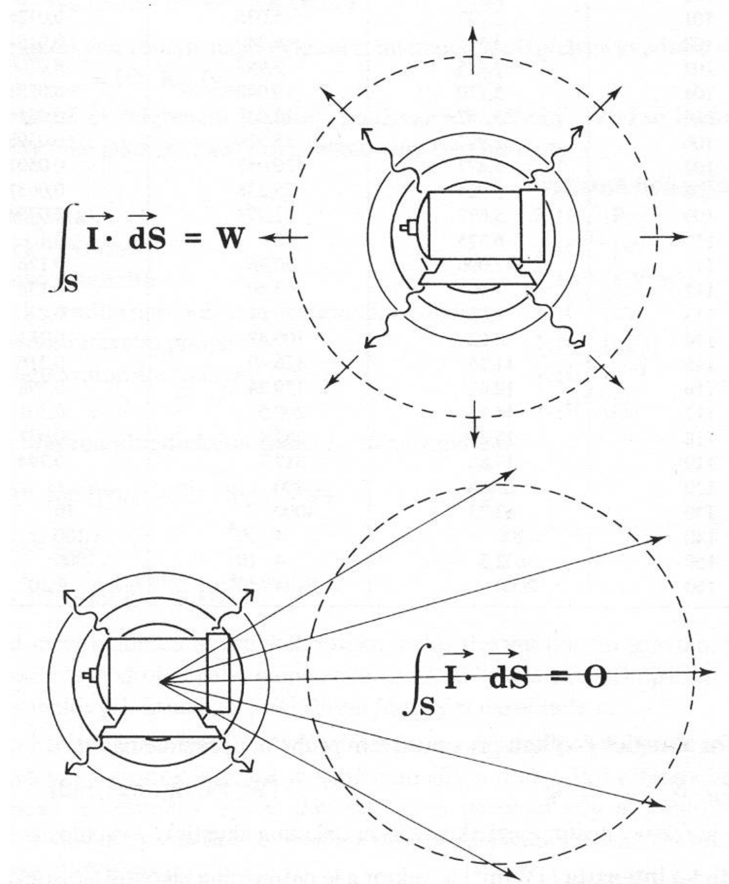
- Measuement using sound intensity (ISO 9614-1,2,3)

2.3.1 Sound power measurement by sound intensity

$$W = \oiint_{(S)} \boldsymbol{I} \cdot \boldsymbol{n} \,\mathrm{d}S$$



Choice of surface defines the source



Approximation of surface integral

$$W = \oiint_{(S)} \boldsymbol{I} \cdot \boldsymbol{n} \, \mathrm{d}S$$

$$W \approx \sum_{i=1}^{N} I_{ni} S_i \qquad W \approx S \frac{1}{T_s} \int_0^{T_s} I_n \mathrm{d}t$$